Rank-Nullity Theorem (Poole 3.26): If A is a $m \times n$, then

$$\operatorname{rank}(A) + \operatorname{nullity}(A) = n \tag{7}$$

Example 8: Consider the matrix $A = \begin{bmatrix} 2 & 1 & 0 & 2 & -1 \\ 1 & 0 & 1 & 2 & 0 \\ 1 & 0 & 1 & 2 & 0 \\ 2 & 1 & 0 & 2 & -1 \end{bmatrix}$. Calculate rank($A$) and nullity($A$).

$\checkmark \ \checkmark \ \times \ \times \ \times$

$\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ is a basis for col(A),

$\operatorname{rank}(A) = \dim(\operatorname{col}(A)) = 2$

$\operatorname{nullity}(A) = n - \operatorname{rank}(A)$
$= 5 - 2$
$= 3$

Note 7: Elementary row operations *preserve* the null space of a matrix and thus *preserve* the *nullity* of a matrix.

Corollary 2: If the $m \times n$ matrices A and C are row equivalent then

$$\operatorname{rank}(A) = \operatorname{rank}(C) \tag{8}$$

Proof: $\operatorname{rank}(A) = n - \operatorname{nullity}(A)$
$= n - \operatorname{nullity}(C)$
$= \operatorname{rank}(C)$ ✓

Note 8: Elementary row operations *do not preserve* the column space of a matrix but *do preserve* the *rank* of a matrix.